

# A no-singularity scenario in loop quantum gravity

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## Abstract

Canonical methods allow the derivation of effective gravitational actions from the behavior of space-time deformations reflecting general covariance. With quantum effects, the deformations and correspondingly the effective actions change, revealing dynamical implications of quantum corrections. A new systematic way of expanding these actions is introduced showing as a first result that inverse-triad corrections of loop quantum gravity simplify the asymptotic dynamics near a spacelike collapse singularity. By generic quantum effects, the singularity is removed.

## 1 Introduction

Quantum gravity, with general relativity as its limit at small gravitational potential and curvature, endows space-time with quantum effects. Depending on the approach used, there are different modifications to the classical dynamical equations. It is difficult to analyze these theories directly except in model systems which often focus on just one type of modification and severely restrict the class of solutions looked for, for instance by symmetry reduction. With such restrictions, on the other hand, it is impossible to say how generic the results are. Here, we employ and extend a new application of an old technique in canonical gravity to evaluate *unrestricted* loop quantum gravity regarding one important issue: the singularity problem.

## 2 Singularities

The singularity problem of general relativity states that space-time is generically incomplete, with only finite ranges of time over which we can extrapolate in some directions. The most infamous example, the big-bang singularity, shows that general relativity does not allow us to extend our solutions to space-time before the big bang. As in this case, singularities often, but not always, are signaled by diverging curvature quantities or, in physical terms, infinite tidal forces and energy densities. It is possible to evade singularities classically [1], but no general mechanism is known to avoid the strict verdict of singularity theorems without fine-tuning.

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At large curvature, where many cases of singularities occur, quantum gravity is expected to be important. It is therefore a common hope that quantum effects, for instance new repulsive contributions to the gravitational force, can help to avoid singularities generically; indications do indeed exist [2, 3] in reduced models especially of loop quantum cosmology [4, 5]. The general mechanism shows that wave functions for the universe, subject to difference equations [6], can be extended through a classical singularity. A more intuitive picture, available in some cases, is a “bounce” [7, 8, 9] of the cosmological scale factor when the energy density is nearly Planckian.<sup>1</sup> This effect, however, makes use of only one quantum effect, called holonomy correction as introduced in more detail below, and implicitly or explicitly ignores several others. Holonomy corrections in isotropic cosmological models bound the energy density  $\rho$  from above by a nearly Planckian upper limit  $\rho_{\text{max}}$ , and correspondingly are strongest when  $\rho/\rho_{\text{max}}$ , or  $\ell_{\text{P}}^2/\ell_{\text{H}}^2$  with the Hubble distance  $\ell_{\text{H}} = 1/\mathcal{H}$ , is of the order one. The same size of corrections is expected from higher-curvature terms (or loop diagrams in perturbative quantum gravity [12]), but these are ignored in most bounce models of loop quantum cosmology. These models, therefore, do not provide generic scenarios.

Moreover, it is non-trivial to extend bounce scenarios to inhomogeneous geometries. Regarding generic inhomogeneous singularities, one often tries to appeal to the Belinskii–Khalatnikov–Lifschitz (BKL) conjecture [13]: Near a spacelike curvature singularity, time derivatives seem to become dominant in the dynamics of general relativity. (Spatial derivatives can be large, but they do not significantly contribute to the evolution.) An expanding or collapsing universe is then essentially a collection of homogeneous cosmological models, one per spatial point. Homogeneous models are much easier to analyze, especially in loop quantum cosmology [3]. If they can be shown to become non-singular by quantum effects, the full BKL-like dynamics may be non-singular.

The applicability of these ideas depends on how singularities are resolved. The asymptotic nature of BKL arguments and the definitive turning point of a bounce do not agree well, making singularity resolution in inhomogeneous situations difficult. Moreover, despite some progress [14, 15, 16] the BKL scenario remains to be proven and fully formulated even classically. Some difficulties concern the fact that the scenario is not obvious from the action or equations of motion but rather follows from an often painstaking analysis of solutions. This fact makes results depend on a preferred time variable, derivatives by which become dominant. Such a choice of time might be natural for an analysis of the present, nearly homogeneous universe, but becomes questionable when assumed near a singularity. It remains unclear whether the BKL scenario can be compatible with general covariance.

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<sup>1</sup>Generically, bounces in loop quantum cosmology are realized with matter whose energy is dominated by its kinetic contribution [10]. For other matter ingredients it is not clear how generically bounces occur since quantum back-reaction — or higher-curvature corrections — remains poorly controlled in those cases [11].

### 3 Off-shell loop quantum gravity

The dependence on what is considered time, or a spatial slice at constant time, points to another important problem, the recognition of which will eventually allow us to make progress. In a canonical formulation of general relativity, space-time covariance is implemented by local symmetries under deformations of spatial slices in space-time [17]. There are three independent deformations generated by  $D[N^i] = \int d^3x N^i(x) D_i(x)$  tangential to space and one,  $H[M] = \int d^3x M(x) H(x)$ , normal to space, with  $D_i(x)$  and  $H(x)$  depending on the spatial metric  $g_{ij}$  and its rate of change. Geometrical considerations of deformations imply that the symmetry generators must satisfy a certain algebra under Poisson brackets; in particular, for  $H[M]$  we have the relationship

$$\{H[M_1], H[M_2]\} = D[\beta(M_1 \vec{\nabla} M_2 - M_2 \vec{\nabla} M_1)] \quad (1)$$

with  $\beta = 1$  classically. That an algebra of this form be realized is not only a crucial condition for the consistency of any space-time theory, including quantum gravity perhaps with quantum corrections  $\beta \neq 1$ ; Eq. (1) is also a key tool for an analysis of the dynamics, for it contains much information: The classical equations of motion (to second derivative order) follow from it [18, 19], and higher derivative orders are restricted to ensure covariance.

However, an analysis of relations such as (1) — and as a consequence the general problem of defining a consistent quantum theory of gravity — is usually complicated by the fact that not only structure constants but also phase-space functions enter, in particular the spatial metric used to define the gradient  $\nabla^i = g^{ij} \partial / \partial x^j$ . In particular, it is not known whether loop quantum gravity [20, 21, 22] is fully consistent, but it has led to consistent deformations (1) with  $\beta \neq 1$  in several model systems and for quantum corrections of different types.

Loop quantum gravity has a dynamics of SU(2)-Yang–Mills form, with a connection  $A_j^I$  and a conjugate field, the densitized triad  $E_I^j$ .<sup>2</sup> To quantize them, these fields are integrated along curves and surfaces, respectively, to obtain holonomies of  $A_j^I$  and fluxes of  $E_I^j$ . Using a U(1)-simplification for the sake of illustration, we write  $A_j^I = c_{(j)}(x) \delta_j^I$  and  $E_I^j = p^{(j)}(x) \delta_j^I$ . Integrated variables, labeled by curves  $e$  along which integrations are done, are  $c_e = \int_e t^j c_j(\lambda) d\lambda$  and  $F_S = \int_S n_j p^j(y) d^2y$  with curves  $e$  tangent to  $t^j$  and surfaces  $S$  co-normal to  $n_j$ . Loop quantum gravity is based on a representation where all holonomies  $h_e = \exp(ic_e)$  act as shift operators on a U(1)-theory per edge  $e$ , and fluxes  $F_S$  act by derivatives with discrete spectra [23].

The specific kinematics implies characteristic corrections on top of the usually expected higher-curvature terms and higher time derivatives in quantum dynamics (which in a canonical quantization follow from quantum back-reaction [24, 25]). The use of holonomies  $h_e$  for  $c_j(x)$  in Hamiltonian operators and in  $H[M]$  implies higher-order corrections by powers of

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<sup>2</sup>To be closer to the notation of [18, 19] used crucially for some derivations, we deviate from the usual practice in loop quantum gravity of denoting tangent-space indices by  $a, b, c, \dots$ . Those indices are labelled  $i, j, k, \dots$ , while  $I, J, K, \dots$  are used for internal indices. We will suppress the Barbero–Immirzi parameter in this article.

$c_j(x)$  or  $\dot{p}^j(x)$ , and also higher spatial derivatives if a derivative expansion for the integrated  $c_e$  is used. Both consequences resemble terms from higher-curvature corrections, but they are not the same because they lack higher time derivatives. Accordingly, holonomy corrections, unlike higher-curvature ones, modify the space-time structure by implying quantum corrections in (1). Several examples for consistent deformations exist in which  $\beta$  is of the form  $\cos(2\delta c_j(x))$  with a parameter  $\delta$  depending on the discreteness scale (the length of curves  $e$  used to construct a state). They include spherical symmetry [26, 27], 2 + 1 gravity [28], and perturbative inhomogeneity [29].

A second type of quantum-geometry corrections is more indirect: Matter Hamiltonians and  $H[M]$  always require inverses of  $p^j(x)$ , but the quantized  $F_S$  with their discrete spectra containing zero do not have inverse operators. Instead, general techniques [30] exist to derive operators with the inverse as their classical limit: Pick intersecting pairs of curves  $e$  and surfaces  $S$ , use Poisson brackets  $\{c_e, F_S\} = 8\pi G$  with the gravitational constant  $G$  and write  $|F_S|^{-1/2}\text{sgn}(F_S) = \{c_e, |F_S|^{1/2}\}/4\pi G = -i(h_e^*\{h_e, |F_S|^{1/2}\} - h_e\{h_e^*, |F_S|^{1/2}\})/8\pi G$ . Loop quantum gravity has quantized  $\hat{h}_e$  as finite shift operators, for which commutator identities show that  $|F_S|^{-1/2}\widehat{\text{sgn}}(F_S) = (|\hat{F}_S + 8\pi\ell_P^2|^{1/2} - |\hat{F}_S - 8\pi\ell_P^2|^{1/2})/8\pi\ell_P^2$  is well-defined (and zero) even on zero-eigenstates of  $\hat{F}_S$  where the classical inverse would diverge. At small flux values, these constructions imply inverse-triad corrections [31] with classical inverses such as  $1/p^j$  replaced by  $\alpha(p^i)/p^j$  for a correction function  $\alpha$  that vanishes at zero  $p^j$ . Consistent deformations (1) with  $\beta = \alpha^2$  have been found in spherical symmetry [26, 32], in 2 + 1 gravity [33], and with perturbative inhomogeneity [34].<sup>3</sup> In some models, holonomy and inverse-triad corrections have been combined, showing multiplicative behavior of the deformation function  $\beta$ . With spherical symmetry, for instance, the combined correction function is  $\beta = \alpha(p^i)^2 \cos(2\delta c_j)$  [27]. Several rather different systems have been analyzed, with various methods (effective techniques and operator calculations). In all cases, deformed algebras of related forms have been found, while undeformed consistent versions do not seem to exist.

The presence of all the corrections makes it hard to keep an overview of the evolution. The situation is much clearer at the level of the algebra (1), whose corrections contained in  $\beta - 1$  appear to be rather universal. With canonical techniques, one can reconstruct an effective action from the deformed algebra for  $\beta \neq 1$ . An expansion by the rate of change of the spatial metric then shows general properties of loop quantum gravity that reveal the space-time structure near a collapse singularity.

## 4 Effective action

To that end, as in [19], we first perform a Legendre transformation from the Hamiltonian  $H(x)$ , depending on  $x$  via the spatial metric  $g_{ij}$  and its momentum  $\pi^{ij}$ , to  $L(x) = \pi^{ij}v_{ij} - H(x)$  with  $v_{ij}(x) = N^{-1}\delta H[N]/\delta\pi^{ij}(x)$  the normal rate of change of  $g_{ij}$ . For derivatives by the spatial metric, we have  $\delta H/\delta g_{ij} = -\delta L/\delta g_{ij}$ . The relation (1), written explicitly

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<sup>3</sup>Also some Wheeler–DeWitt models have been argued to give rise to deformed hypersurface-deformation algebras [35].

with a Poisson bracket  $\{f, g\} = \int d^3x (\delta f / \delta g_{ij}(x)) (\delta g / \delta \pi^{ij}(x)) - (f \leftrightarrow g)$ , then implies  $v_{ij}(x') \delta L(x) / \delta g_{ij}(x) + \beta D^i \nabla_i \delta(x, x') - (x \leftrightarrow x') = 0$ . A metric changes under spatial deformations such that  $D^i = -2 \nabla_j \pi^{ij}$ , and after replacing  $\pi^{ij}$  with  $\delta L / \delta v_{ij}$  we obtain a linear equation

$$\frac{\delta L(x)}{\delta g_{ij}(x')} v_{ij}(x') \delta(x, x') - (x \leftrightarrow x') = -2 \frac{\delta L}{\delta v_{ij}} \nabla_j (\beta \nabla_i \delta(x, x')) - (x \leftrightarrow x'). \quad (2)$$

We can solve this equation order by order in an expansion by  $v_{ij}$ , writing

$$L(x) = \sum_{n=0}^{\infty} \sum_{N_1, \dots, N_n=0}^{\infty} L^{(i_1, j_1, k_1^{(1)}, \dots, k_1^{(N_1)}), \dots, (i_n, j_n, k_n^{(1)}, \dots, k_n^{(N_n)})}(g_{ij}) \quad (3)$$

$$\times (\nabla_{k_1^{(1)}} \cdots \nabla_{k_1^{(N_1)}} v_{i_1 j_1}) \cdots (\nabla_{k_n^{(1)}} \cdots \nabla_{k_n^{(N_n)}} v_{i_n j_n})$$

and an analogous expansion for  $\beta(x)$ . To be specific, we will assume that only even terms appear in (3), to ensure time-reversal invariance. (If the assumption is violated, our arguments still go through.) We deviate from [19] because of the presence of  $\beta$  as well as higher spatial derivatives in the expansion. The former is a consequence of quantum space-time with a deformed relation (1), and the latter allows for non-local effects in holonomies and fluxes. Moreover, unlike [19] which derived the classical action to second order in  $v_{ij}$  and in derivatives, we are interested in violent near-singular regimes in which many terms of the expansion may be relevant.

We focus on  $\beta^0(g_{ij})$ , the leading coefficient in the expansion of  $\beta$  independent of  $v_{ij}$ . From the form of consistent deformations known, this coefficient is sensitive to inverse-triad corrections, and it becomes very small near a classical spacelike collapse singularity where fluxes are zero [31]. Coefficients of  $\beta$  of higher order in the  $v$ -expansion, on the other hand, are determined by all types of quantum corrections and do not have characteristic values.

Solving (2) order by order as a recurrence relation for coefficients in (3) shows that terms of order  $n + 2$  in  $v_{ij}$  and its spatial derivatives are related to terms of lower order  $n$  *divided by*  $\beta^0$ : Take the coefficients of order  $n + 1$  in the  $v$ -expansion of all terms in (2), obtained by expanding  $L$  and  $\beta$  and collecting terms of the same order in  $v_{ij}$  and its spatial derivatives. The highest-order  $L$ -coefficient, of order  $n + 2$ , is obtained from  $v_{ij}$ -derivative terms on the right-hand side with only the leading coefficient  $\beta^0$  of  $\beta$  (or  $\nabla_j \beta^0$  which too is small for small fluxes) as a factor. Higher orders in  $\beta$  are multiplied with  $L$ -coefficients of order at most  $n$  so as to produce a total order of  $n + 1$ . On the left-hand side of the equation, order  $n + 1$  is obtained with  $L$ -coefficients of order  $n$ . The  $L$ -coefficient of order  $n + 2$  multiplied with  $\beta^0$  then equals a combination of  $L$ -coefficients of order at most  $n$  and without a factor of  $\beta^0$ .

To second order in the derivative expansion, for instance, we have the effective action  $L_2 = (16\pi G)^{-1} \sqrt{\det g} \left( \text{sgn}(\beta^0) |\beta^0|^{-1/2} \mathcal{G}^{ijkl} v_{ij} v_{kl} + \sqrt{|\beta^0|} {}^{(3)}R - 2\Lambda \right)$  with  $\mathcal{G}^{ijkl} = \frac{1}{4} (g^{i(k} g^{l)j} - g^{ij} g^{kl})$  and the spatial Ricci scalar  ${}^{(3)}R$  [36]. Continuing the expansion to higher orders and using our recurrence arguments then shows that a term of order  $n$  in the  $v$ -expansion is multiplied with  $|\beta^0|^{(1-n)/2}$ . Terms with higher powers of  $1/\beta^0$  play the largest role near a

spacelike singularity, while others can be ignored. But also the typical size of products of  $v_{ij}$  and its spatial derivatives matters, which could compensate for suppression factors of  $\beta^\emptyset$ .

## 5 Derivative expansion

In order to see what the dominant terms are generically, we combine the expansion in  $v_{ij}$  with a derivative expansion. The  $v$ -expansion by  $n$  in (3), in which we consider  $v_{ij}$  and its spatial derivatives as being of the same order, is suggested by the form of (2) with its different dependences on  $v_{ij}$ . The  $v$ -order is different from the derivative order, which is useful to tell what terms are generically of similar orders in a strong-curvature regime. In contrast to the  $v$ -expansion, the derivative expansion does not distinguish between space and time derivatives. One factor of  $v_{ij}$  in our expansion contributes one derivative order because  $v_{ij}$ , as the rate of change of  $g_{ij}$ , has one time derivative. Including spatial derivatives, terms of fixed derivative order  $N$  are of the form

$$\begin{aligned} & |\beta^\emptyset|^{(1-N)/2} v^N + |\beta^\emptyset|^{(2-N)/2} (v^{N-1} g' + v^{N-2} v') \\ & + |\beta^\emptyset|^{(3-N)/2} (v^{N-2} (g'' + (g')^2) + v^{N-3} (v'' + v' g') + v^{N-4} (v')^2) + \dots \end{aligned} \quad (4)$$

where we do not spell out individual coefficients and indices. All terms have the same number of derivatives, but different numbers of  $v$ -factors as indicated by the powers of  $\beta^\emptyset$ . Terms of equal numbers of derivatives are generically of the same size in a given curvature regime, and factors of  $\beta^\emptyset$  decide which terms are important for the dynamics. For small  $\beta^\emptyset$ , the leading term,  $|\beta^\emptyset|^{(1-N)/2} v^N$ , is dominant in the expansion (4), and it does not contain any spatial derivatives.

Near a spacelike singularity, the effective action therefore contains only time derivatives, obeying the dynamics of some homogeneous model. The picture is reminiscent of the BKL scenario, but in the form found here, making use of quantum effects, it has several advantages. First, the dominance of time derivatives follows from a consideration of the effective action, without the need to solve equations of motion. Secondly, we do not make use of a preferred time coordinate or spatial slicing of space-time. Our considerations are covariant because they are based on a consistent deformation (1) of the classical covariance algebra. It is the quantum-corrected covariance algebra itself which implies the suppression of spatial derivatives, not the choice of a preferred slicing that would break covariance. Finally, our arguments remain valid at a general level of perturbative quantum gravity, referring to all orders in the curvature expansion.

## 6 No singularities

We are now ready to apply our new scenario to the singularity problem. Imagine that we follow the evolution of some quantum geometry toward a spacelike collapse singularity. Effective equations describe the dynamics of states parameterized by expectation values

and moments, as well as quantum-geometry corrections. When curvature grows large, different effects become important. They may stop the collapse or trigger a bounce, in which case the singularity is not reached.<sup>4</sup> But if there is nothing to stop the approach to vanishing fluxes, which classically amounts to a singularity, inverse-triad corrections will make  $\beta^0$  smaller and smaller. By our new scenario, time derivatives in the effective action then dominate and we can continue the evolution by using homogeneous mini-superspace models. At some point, effective actions will break down or all terms in the derivative expansion will have to be considered. But instead of doing this, we can use exact quantizations of homogeneous models in loop quantum cosmology and conclude, following [2, 3], that they are non-singular: any quantum state is extended uniquely across the classical singularity. Once we extend the state through vanishing fluxes, effective actions will again be available and continue our evolution.

Generically, no collapse singularity occurs in loop quantum gravity, even for geometries not required to obey any symmetry. The key mechanism is inverse-triad corrections and the new space-time structure they imply via (1). These corrections both provide the suppression of spatial derivatives in effective actions and facilitate the extension of wave functions across a classical singularity.

Our treatment does not apply to all space-time metrics because one could choose a time coordinate and initial values such that time and spatial derivatives are unbalanced, with spatial ones unnaturally large compared to time ones. However, this would require fine-tuning and provide a negligible subset of universe models. And changing the space-time slicing would bring one back to a situation in which our arguments do apply. Only the expansion is more difficult to organize in some slicings; our mechanism of singularity resolution, which is covariant, is always realized.

## 7 Conclusions

In spite of its generality, our scenario is not a complete proof of the absence of singularities in loop quantum gravity, and we refrain from calling it a “no-singularity theorem.” We must assume that loop quantum gravity can be consistent, obeying local symmetries of a covariance algebra of the form (1) with some function  $\beta$ . Good evidence for this behavior exists from several model systems [34, 27, 26, 28, 32, 29, 33], and no undeformed off-shell algebra has been found, but a demonstration in general terms seems currently out of reach. Our arguments then show that any consistent realization is generically non-singular. They also highlight the importance of off-shell properties of canonical quantum gravity, whose daunting derivations are often avoided by resorting to gauge-fixing or deparameterization before quantization, leaving the quantum system contaminated with potentially inconsistent gauge artefacts.

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<sup>4</sup>As a further consequence of deformed algebras, holonomy corrections do not imply a true bounce, as it seems in some isotropic models [8], but rather trigger signature change since  $\beta$  turns negative at near-Planckian curvature [36]. If evolution does not stop by holonomy effects and inverse-triad corrections become large, we always have  $\beta^0 > 0$ .

We have provided not only the first general statement about non-singular evolution in quantum gravity; we have also shown how dynamical consequences can be derived taking into account all expected corrections of quantum gravity. Holonomy corrections and higher-curvature corrections both contribute to higher orders in the  $v$ -expansion and can have significant effects, but they leave the leading coefficient  $\beta^0$  unchanged and therefore do not interfere with the resolution of singularities. For inverse-triad corrections, we have explicitly displayed expressions obtained with a  $U(1)$ -simplification. With the non-abelian  $SU(2)$ , some expressions are more complicated, providing an additional dependence on  $v_{ij}$  [37]: holonomies in terms such as  $\hat{h}_e[\hat{h}_e^\dagger, |\hat{F}_S|^{1/2}]$  no longer cancel completely. Also these contributions can be taken care of by changing higher orders in the  $v$ -expansion, without affecting  $\beta^0$ .

In addition to solving the singularity problem in loop quantum gravity, our methods have several other consequences. By relating inhomogeneous to homogeneous evolution in general terms, they show how (and what) minisuperspace effects can be used for general geometries, and how reliable they are. We see that especially inverse-triad effects, with their characteristic form, survive even in regimes in which curvature is large and other quantum effects play equally important roles. We also provide a new scenario for the approach of space-time toward a classical singularity, which is similar in spirit to the BKL picture but relies crucially on quantum effects and avoids several downsides of the BKL arguments. Given the influence that the BKL scenario has had over the past few decades, we expect that our new scenario will play an important role for further analysis of the dynamics of loop quantum gravity.

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